Halloween maths activities for IGCSE and GCSE: SL / AS

IGCSE Math Revision · IB Maths SL and HL (AA and AI) · 31 October · Student sheet

Name _____ Class _____ Date _____

The starter is non-calculator. A GDC is useful for the main activity.

Starter (5 minutes)

1. Solve $2^x = 32$.

2. Find the value of $\log_3 81$.

3. Events A and B are independent, with P(A) = 0.3 and P(B) = 0.5. Find P(A ∩ B).

4. Find the distance between the points (1, 2) and (4, 6).

Main activity (35 minutes)

Task A: Zombie outbreak: an exponential model

The number of zombies t hours after midnight on 31 October is modelled by $Z(t) = 20e^{0.35t}$.

a. Write down the number of zombies at midnight.

b. Find the number of zombies after 5 hours, to the nearest whole number.

c. Find the time when there are 1000 zombies. Give your answer to 3 significant figures.

d. Find the time it takes for the number of zombies to double.

Task B: A better model: the logistic limit

A village has 2000 people, so the zombie numbers cannot grow for ever. A better model is $Z(t) = 2000 \div (1 + 99e^{-0.5t})$. *HL; SL students can do every part with logarithms.*

a. Find $Z(0)$.

b. What happens to $Z(t)$ as t gets very large? Explain what this means.

c. Find the time when half the village are zombies, to 3 significant figures.

d. Find $Z(10)$ to 3 significant figures.

Task C: Pumpkin geometry

A pumpkin is modelled as a sphere of radius 15 cm.

a. Find the volume of the pumpkin, to 3 significant figures.

b. Find its surface area, to 3 significant figures.

c. Find $\mathrm{d}V/\mathrm{d}r$ when $r = 15$. What do you notice?

Task D: Trick or treat

60% of houses on a street have a pumpkin outside. At a house with a pumpkin, the probability of a treat is 0.9; at a house without, it is 0.4.

a. Find the probability that a house chosen at random gives a treat.

b. Given that a house gave a treat, find the probability that it had a pumpkin.

c. You visit 10 houses chosen at random. Find the probability of at least 8 treats, to 3 significant figures.

Extension

1. Early in an outbreak, the logistic model $Z(t) = 2000 \div (1 + 99e^{-0.5t})$ is close to the exponential model $E(t) = 20e^{0.5t}$. Find the percentage error in using E instead of Z when $t = 2$, to 3 significant figures.

2. (HL) A ghost floats along the line $\mathbf{r} = (1, 2, 0) + \lambda(2, -1, 2)$, where the floor of the haunted house is the plane $z = 0$. (a) Show that the ghost passes through the top of the chimney at $(7, -1, 6)$. (b) Find the angle between the ghost's path and the floor.