Halloween maths activities for IGCSE and GCSE: SL / AS: answers

IGCSE Math Revision · IB Maths SL and HL (AA and AI) · 31 October · Teacher answers and notes

The starter is non-calculator. A GDC is useful for the main activity.

Teacher notes

- Ask the class to compare the two zombie models on one GDC screen: the curves agree early on and then separate. That is the whole point of a logistic model.
- Part (c) of the pumpkin task links the derivative of volume to surface area: a nice discussion of why $\mathrm{d}V/\mathrm{d}r = S$ for a sphere.
- In the trick-or-treat task, students often divide by 0.6 instead of 0.7 for the conditional probability. A tree diagram on the board fixes it.

Starter

1. **$x = 5$**
   - $32 = 2^5$, so $x = 5$.
2. **4**
   - $3^4 = 81$, so $\log_3 81 = 4$.
3. **0.15**
   - For independent events, $P(A \cap B) = P(A) \times P(B) = 0.3 \times 0.5 = 0.15$.
4. **5**
   - $\sqrt{(3^2 + 4^2)} = \sqrt{25} = 5$

Task A: Zombie outbreak: an exponential model

a. **20**
- $Z(0) = 20e^0 = 20$

b. **115**
- $20e^{1.75} = 20 \times 5.7546\ldots = 115.09\ldots$
- 115 zombies

c. **11.2 hours**
- $20e^{0.35t} = 1000$, so $e^{0.35t} = 50$.
- $0.35t = \ln 50$, so $t = \ln 50 \div 0.35 = 11.177\ldots$
- 11.2 hours

d. **1.98 hours**
- $e^{0.35t} = 2$, so $t = \ln 2 \div 0.35 = 1.980\ldots$
- 1.98 hours (3 s.f.)

Task B: A better model: the logistic limit

a. **20**
- $2000 \div (1 + 99) = 20$

b. **It approaches 2000: the whole village**
- As $t \to \infty$, $e^{-0.5t} \to 0$, so $Z \to 2000 \div 1 = 2000$.
- The model says the number of zombies levels off at the size of the village.

c. **9.19 hours**

- $2000 \div (1 + 99e^{-0.5t}) = 1000$ gives $99e^{-0.5t} = 1$.
- $-0.5t = \ln(1/99)$, so $t = 2 \ln 99 = 9.190\ldots$
- 9.19 hours

d. **1200**

- $2000 \div (1 + 99e^{-5}) = 2000 \div 1.66706\ldots = 1199.7\ldots$
- 1200 (3 s.f.)

Task C: Pumpkin geometry

a. **14 100 cm³**

- $V = \frac{4}{3}\pi \times 15^3 = 4500\pi = 14\,137.1\ldots$
- 14 100 cm^3

b. **2830 cm²**

- $S = 4\pi \times 15^2 = 900\pi = 2827.4\ldots$
- 2830 cm^2

c. **900π ≈ 2830 cm³ per cm: the same as the surface area**

- $V = \frac{4}{3}\pi r^3$, so $dV/dr = 4\pi r^2$.
- At $r = 15$ this is 900π, which is exactly the surface area: a thin extra skin of thickness δr adds about $S \times \delta r$ to the volume.

Task D: Trick or treat

a. **0.7**

- $0.6 \times 0.9 + 0.4 \times 0.4 = 0.54 + 0.16 = 0.7$

b. **27/35 ≈ 0.771**

- $P(\text{pumpkin} \mid \text{treat}) = 0.54 \div 0.7 = 27/35 = 0.771\ldots$

c. **0.383**

- $X \sim B(10, 0.7)$.
- $P(X \geq 8) = P(8) + P(9) + P(10) = 0.2335 + 0.1211 + 0.0282 = 0.3828\ldots$
- 0.383

Extension

1. **1.72%**

- $E(2) = 20e = 54.366\ldots$; $Z(2) = 2000 \div (1 + 99e^{-1}) = 53.445\ldots$
- Percentage error = $(54.366 - 53.445) \div 53.445 \times 100 = 1.72\%$

2. **(a) λ = 3 (b) 41.8°**

- (a) $\lambda = 3$ gives $(1 + 6, 2 - 3, 0 + 6) = (7, -1, 6)$.
- (b) The floor has normal $(0, 0, 1)$. $\sin\theta = |2| \div (\sqrt{(4 + 1 + 4)} \times 1) = 2/3$.
- $\theta = \arcsin(2/3) = 41.8°$

Every question is original and every answer was checked before publishing.