Halloween maths activities for IGCSE and GCSE: HL / Further

IGCSE Math Revision · IB HL and A Level Further Maths (no calculator) · 31 October · Student sheet

Name _____ Class _____ Date _____

No calculator needed: every answer is exact.

Starter (5 minutes)

1. Write $(1 + \mathrm{i})^4$ in the form $a + b\mathrm{i}$.

2. Find the sum of the whole numbers from 1 to 31, one for each day of October.

3. Find the exact value of $\int_0^{\ln 2} \mathrm{e}^x \, \mathrm{d}x$.

4. In how many ways can 5 different masks be hung in a row?

Main activity (30 minutes)

Task A: Zombie logistic

A zombie population Z (in thousands) grows so that $\mathrm{d}Z/\mathrm{d}t = Z(1 - Z/10)$, with $Z = 1$ when $t = 0$ (t in days).

a. Find the rate of growth when $Z = 4$.

b. For which value of Z is the population growing fastest?

c. Show that $Z = 10/(1 + 9\mathrm{e}^{-t})$ satisfies the differential equation and the starting value. What happens to Z as $t \to \infty$?

d. Find the exact time at which $Z = 5$.

Task B: Cauldron roots

The witches' magic number z satisfies $z^3 = 8\mathrm{i}$.

a. Write 8i in modulus–argument form.

b. Find the three cube roots of 8i, giving each in the form $a + b\mathrm{i}$.

c. The three roots are the corners of a triangle on the Argand diagram. Find its exact area.

Extension

1. Prove by induction that $1 + 3 + 5 + \ldots + (2n - 1) = n^2$ (the sweets in an n by n tray, built up in L shapes). How many odd numbers, starting from 1, add up to 961?

2. A haunted die is biased so that $\mathrm{P}(X = k) = k/21$ for $k = 1, 2, \ldots, 6$. Find $\mathrm{E}(X)$.