Halloween maths activities for IGCSE and GCSE: HL / Further: answers

IGCSE Math Revision · IB HL and A Level Further Maths (no calculator) · 31 October · Teacher answers and notes

No calculator needed: every answer is exact.

Teacher notes

- For IB HL and A Level Further Maths classes. Every answer is exact and the whole set works without a calculator.
- Task A pairs well with a sketch of dZ/dt against Z: ask where the population curve is steepest before any algebra.

Starter

1. **−4 (that is, −4 + 0i)**
   - $(1 + i)^2 = 2i$
   - $(2i)^2 = -4$
2. **496**
   - $31 \times 32 \div 2 = 496$
3. **1**
   - $[e^x]$ from 0 to $\ln 2 = 2 - 1 = 1$
4. **120**
   - $5! = 120$

Task A: Zombie logistic

a. **12/5 = 2.4 thousand zombies per day**
   - $4 \times (1 - 4/10) = 4 \times 3/5 = 12/5$

b. **$Z = 5$**
   - The growth rate is $Z - Z^2/10$.
   - Its derivative with respect to Z is $1 - Z/5$, which is 0 when $Z = 5$ (a maximum: the second derivative is $-1/5$).

c. **$Z \to 10$ (10 000 zombies)**
   - At $t = 0$, $Z = 10/10 = 1$.
   - $dZ/dt = 90e^{-t}/(1 + 9e^{-t})^2$, and $Z(1 - Z/10) = 10/(1 + 9e^{-t}) \times 9e^{-t}/(1 + 9e^{-t})$ is the same.
   - As $t \to \infty$, $e^{-t} \to 0$, so $Z \to 10$.

d. **$t = \ln 9 = 2 \ln 3$ days**
   - $10/(1 + 9e^{-t}) = 5$ gives $9e^{-t} = 1$.
   - $e^t = 9$, so $t = \ln 9$

Task B: Cauldron roots

a. **8 cis(π/2)**
   - $|8i| = 8$, and $8i$ lies on the positive imaginary axis, so its argument is $\pi/2$.

b. **√3 + i, −√3 + i and −2i**
   - The roots are $2\,\mathrm{cis}(\pi/6 + 2k\pi/3)$ for $k = 0, 1, 2$.
   - $2\,\mathrm{cis}(\pi/6) = \sqrt{3} + i$, $2\,\mathrm{cis}(5\pi/6) = -\sqrt{3} + i$, $2\,\mathrm{cis}(3\pi/2) = -2i$

c. **3√3**
   - The roots are equally spaced on a circle of radius 2, so the triangle is equilateral.
   - Area $= 3 \times \tfrac{1}{2} \times 2 \times 2 \times \sin(2\pi/3) = 6 \times \sqrt{3}/2 = 3\sqrt{3}$

Extension

1. **31**
   - Proof: true for $n = 1$; if the first k add to k^2, adding $2k + 1$ gives $(k + 1)^2$.
   - $n^2 = 961$, so $n = 31$
2. **13/3**
   - $E(X) = \Sigma\, k \times k/21 = (1 + 4 + 9 + 16 + 25 + 36)/21$
   - $= 91/21 = 13/3$

Every question is original and every answer was checked before publishing.