Fibonacci Day maths activities for IGCSE and GCSE: SL / AS

IGCSE Math Revision · IB Maths SL and HL (AA and AI) · 23 November · Student sheet

Name _____ Class _____ Date _____

The starter is non-calculator. A GDC helps in tasks A and B.

Starter (5 minutes)

1. Write down F_3 to F_{10}.

2. Solve $x^2 = x + 1$, giving exact answers.

3. Work out $F_1 + F_2 + \ldots + F_6$. Compare your answer with F_8.

4. Work out $21 \div 13$ to 3 decimal places.

Main activity (35 minutes)

Task A: The limit of the ratios

Let $r_n = F_{n+1} \div F_n$.

a. Find r_1, r_2, …, r_8, to 3 decimal places where needed. Describe what happens.

b. Divide $F_{n+2} = F_{n+1} + F_n$ by F_{n+1} to show that $r_{n+1} = 1 + 1/r_n$. Assuming r_n tends to a limit φ, find φ exactly and to 3 decimal places.

c. Use $\varphi^2 = \varphi + 1$ to write φ^3 and φ^4 in the form $a\varphi + b$. What do you notice about the numbers?

Task B: Golden rectangles and the spiral

A golden rectangle has its sides in the ratio $\varphi : 1$, where $\varphi = (1 + \sqrt{5})/2$.

a. A 1 by 1 square is cut from a golden rectangle measuring φ by 1, leaving a 1 by $(\varphi - 1)$ rectangle. Show that this smaller rectangle is also golden.

b. A picture frame is a golden rectangle with shorter side 30 cm. Find the longer side to 3 significant figures.

c. Squares with sides 1, 1, 2, 3, 5, 8 and 13 cm fit together to make one rectangle. Find the total area of the squares, and show it equals the area of a 13 cm by 21 cm rectangle.

d. A quarter circle is drawn in each square of part (c), with radius equal to the square's side, to make a spiral. Find the length of the spiral, exactly and to 3 significant figures.

Task C: Fibonacci identities

Spot each pattern, check it, then use it.

a. Find $F_1 + F_2 + \ldots + F_{10}$ and show it equals $F_{12} - 1$.

b. Work out $F_{n-1}F_{n+1} - F_n^2$ for $n = 2, 3, 4, 5$ and 6. Suggest a general result.

c. Every third Fibonacci number is even. How many of $F_1, F_2, \ldots, F_{30}$ are even?

Extension

1. Binet's formula says $F_n = (\varphi^n - \psi^n)/\sqrt{5}$. Check that it gives $F_1 = 1$ and $F_2 = 1$, then use it (with a GDC) to find F_{20}.

2. Prove by induction that $F_1 + F_2 + \ldots + F_n = F_{n+2} - 1$ for every positive integer n.