

Fibonacci Day maths activities for IGCSE and GCSE: answers

IGCSE Math Revision · IGCSE and GCSE (Higher and Extended) · 23 November · Teacher answers and notes

The starter is non-calculator. A calculator is allowed in the main activity.

Teacher notes

- Fibonacci Day is 23 November because 11/23, written month first, reads 1, 1, 2, 3.
- Task A (c) is simultaneous equations in disguise: let students find the general terms first.
- Task C works well on squared paper with compasses: the finished spirals make a good display.

Starter

1. **13, 21, 34**
 - $5 + 8 = 13$, $8 + 13 = 21$, $13 + 21 = 34$
2. **18, 29**
 - $7 + 11 = 18$, $11 + 18 = 29$
3. **$6 + 2\sqrt{5}$**
 - $1 + 2\sqrt{5} + 5 = 6 + 2\sqrt{5}$
4. **$1.6 : 1$**
 - $8 \div 5 = 1.6$

Task A: Fibonacci-type sequences

- a. **31**
 - 2, 5, 7, 12, 19, 31
- b. **$b = 11$**
 - $a, b, a + b, a + 2b, 2a + 3b$
 - $8 + 3b = 41$, so $3b = 33$ and $b = 11$.
- c. **4 and 6**
 - The terms are $a, b, a + b, a + 2b, 2a + 3b, 3a + 5b$.
 - $a + b = 10$ and $3a + 5b = 42$.
 - Substitute $b = 10 - a$: $3a + 50 - 5a = 42$, so $a = 4$ and $b = 6$.

Task B: The golden ratio

- a. **1.625, 1.615, 1.619: they get closer to about 1.618**
 - $13 \div 8 = 1.625$
 - $21 \div 13 = 1.6153\dots$
 - $34 \div 21 = 1.6190\dots$
 - The answers go up and down, getting closer to 1.618...
- b. **1.618**
 - $x^2 - x - 1 = 0$, so $x = (1 \pm \sqrt{5})/2$.
 - The positive solution is $(1 + \sqrt{5})/2 = 1.618$ (3 d.p.).

c. **32.4 cm**

- $20 \times 1.6180... = 32.36...$
- 32.4 cm (1 d.p.)

Task C: Fibonacci squares and the spiral

a. **13 cm by 8 cm**

- The last square is 8 by 8 and sits beside a 5 by 8 rectangle made from the others, so the rectangle is 8 by $(8 + 5) = 8$ by 13.

b. **104 cm²**

- $1 + 1 + 4 + 9 + 25 + 64 = 104$
- $8 \times 13 = 104$

c. **31.4 cm**

- Each quarter circle is $\frac{1}{4} \times 2\pi r = \pi r/2$.
- Total = $\pi/2 \times (1 + 1 + 2 + 3 + 5 + 8) = \pi/2 \times 20 = 10\pi = 31.41...$
- 31.4 cm (3 s.f.)

Extension

1. **Both sides equal $(3 + \sqrt{5})/2$**

- $\phi^2 = (1 + \sqrt{5})^2/4 = (6 + 2\sqrt{5})/4 = (3 + \sqrt{5})/2$.
- $\phi + 1 = (1 + \sqrt{5})/2 + 2/2 = (3 + \sqrt{5})/2$.

2. **20**

- odd + odd = even, odd + even = odd, even + odd = odd, so the pattern odd, odd, even repeats.
- $60 \div 3 = 20$ even numbers.

Every question is original and every answer was recomputed by computer before publishing.