Bonfire Night maths activities for IGCSE and GCSE: SL / AS: answers

IGCSE Math Revision · A Level Maths (Year 12 and Year 13) · 5 November · Teacher answers and notes

The starter is non-calculator. A calculator is needed for the main activity. Take g = 9.8 m s^{-2}.

Teacher notes

- Enjoy fireworks at an organised display; this pack is about the maths only.
- Keep exact values (35 sin 60° = 35√3/2) until the last line: early rounding changes the third significant figure in A (d).
- Ask the class what the model ignores (air resistance, the rocket's own thrust) and how each would change the answers.

Starter

1. **210**
   - $S_{10} = 10/2 \times (2 \times 3 + 9 \times 4) = 5 \times 42 = 210$
2. **16**
   - $a = 8$, $r = ½$: $8 \div (1 - ½) = 16$
3. **10/7 s ≈ 1.43 s**
   - $v = u - gt = 0$ gives $t = 14 \div 9.8 = 10/7$.
4. **$t = 0$ or 4**
   - $4.9t(t - 4) = 0$

Task A: The firework projectile

a. **3.09 s; 46.9 m**
- Vertical: $u = 35 \sin 60° = 30.31...$ m s^{-1}.
- Time to the top: $30.31 \div 9.8 = 3.09$ s.
- Height: $(35 \sin 60°)^2 \div (2 \times 9.8) = 918.75 \div 19.6 = 46.875$, so 46.9 m.

b. **54.1 m**
- Horizontal speed: $35 \cos 60° = 17.5$ m s^{-1}.
- $17.5 \times 3.0929... = 54.13$ m

c. **108 m**
- Flight time is twice the time to the top: 6.186 s.
- $17.5 \times 6.186 = 108.25...$, so 108 m.

d. **3.71 s**
- $30.311t - 4.9t^2 = 30$ gives $4.9t^2 - 30.311t + 30 = 0$.
- $t = 1.237$ or 4.949.
- Time above 30 m = $4.949 - 1.237 = 3.71$ s

Task B: The rocket motor (Year 13)

a. **$t = 4$; 48 m s^{-1}**
- $a = \mathrm{d}v/\mathrm{d}t = 24 - 6t = 0$ when $t = 4$.
- $v(4) = 96 - 48 = 48$

b. **256 m**
- $\int_0^8 (24t - 3t^2)\,\mathrm{d}t = [12t^2 - t^3]_0^8 = 768 - 512 = 256$

Task C: Remember, remember: series

a. **62; 670**
- $u_{20} = 5 + 19 \times 3 = 62$
- $S_{20} = 20/2 \times (5 + 62) = 670$

b. **25 minutes**
- $n/2 \times (10 + 3(n - 1)) \geq 1000$ gives $3n^2 + 7n - 2000 \geq 0$.
- $n \geq (-7 + \sqrt{24\,049}) \div 6 = 24.68...$
- So 25 minutes ($S_{24} = 948$ is not enough).

c. **3905**
- $\Sigma_{r=1}^{5} 5^r = 5(5^5 - 1) \div 4 = 5 \times 3124 \div 4 = 3905$

d. **9 rounds**
- $5(5^n - 1) \div 4 > 1\,000\,000$ gives $5^n > 800\,001$.
- $n > \log 800\,001 \div \log 5 = 8.45...$, so 9 rounds.

Extension

1. **$y = \sqrt{3}x - (2/125)x^2$**
   - $x = 17.5t$, so $t = x \div 17.5$.
   - $y = 30.31t - 4.9t^2 = (35 \sin 60° \div 17.5)x - 4.9x^2 \div 306.25$.
   - $35 \sin 60° \div 17.5 = \sqrt{3}$ and $4.9 \div 306.25 = 0.016 = 2/125$.
2. **6.79 s; 119 m**
   - $-20 = 30.311t - 4.9t^2$ gives $4.9t^2 - 30.311t - 20 = 0$.
   - $t = (30.311 + \sqrt{(918.75 + 392)}) \div 9.8 = 6.787...$
   - $17.5 \times 6.787 = 118.8$, so 119 m.

Every question is original and every answer was checked before publishing.