Bonfire Night maths activities for IGCSE and GCSE: HL / Further

IGCSE Math Revision · IB HL and A Level Further Maths (no calculator) · 5 November · Student sheet

Name _____ Class _____ Date _____

No calculator needed: every answer is exact.

Starter (5 minutes)

1. A rocket's velocity is $3\mathbf{i} + 4\mathbf{j}$ m/s. Find its speed.

2. Solve $\mathrm{d}y/\mathrm{d}t = -2y$ with $y = 5$ at $t = 0$, and find y when $t = \ln 2$.

3. Find the sum to infinity of $1 + 1/3 + 1/9 + \ldots$

4. Find the coefficient of x^2 in the expansion of $(1 + x)^{-1}$.

Main activity (30 minutes)

Task A: Rocket with drag

After its fuel runs out, a rocket's upward velocity v m/s satisfies $\mathrm{d}v/\mathrm{d}t = -10 - v/5$, with $v = 50$ when $t = 0$.

a. Solve the equation to show that $v = 100\mathrm{e}^{-t/5} - 50$. Find v when $t = 5\ln(5/4)$.

b. Find the exact time at which the rocket stops rising.

c. Find the height it gains from the moment the fuel runs out until it stops rising, in the form $a - b\ln 2$.

Task B: Catherine wheel

A spark on the rim of a Catherine wheel has position $\mathbf{r} = 3\cos t\,\mathbf{i} + 3\sin t\,\mathbf{j}$ metres at time t seconds.

a. Find the speed of the spark.

b. Find the magnitude of its acceleration.

c. At $t = \pi/2$ a spark flies off and moves in a straight line with the velocity it had at that moment. Where is it 1 second later?

Extension

1. Prove by induction that $1 \times 2 + 2 \times 3 + \ldots + n(n+1) = n(n+1)(n+2)/3$. Use it to find $1 \times 2 + 2 \times 3 + \ldots + 10 \times 11$.