Bonfire Night maths activities for IGCSE and GCSE: HL / Further: answers

IGCSE Math Revision · IB HL and A Level Further Maths (no calculator) · 5 November · Teacher answers and notes

No calculator needed: every answer is exact.

Teacher notes

- For IB HL and A Level Further Maths classes. Every answer is exact and the whole set works without a calculator.
- In task A, compare with no air resistance ($\mathrm{d}v/\mathrm{d}t = -10$): the rocket would rise for 5 s and gain 125 m.

Starter

1. **5 m/s**
   - $\sqrt{(9 + 16)} = 5$
2. **5/4**
   - $y = 5\mathrm{e}^{-2t}$
   - At $t = \ln 2$: $5\mathrm{e}^{-2\ln 2} = 5/4$
3. **3/2**
   - $1/(1 - 1/3) = 3/2$
4. **1**
   - $(1 + x)^{-1} = 1 - x + x^2 - \ldots$

Task A: Rocket with drag

a. **30 m/s**

- Integrating factor $\mathrm{e}^{t/5}$: $\mathrm{d}(v\mathrm{e}^{t/5})/\mathrm{d}t = -10\mathrm{e}^{t/5}$, so $v\mathrm{e}^{t/5} = -50\mathrm{e}^{t/5} + C$, and $C = 100$.
- At $t = 5\ln(5/4)$, $\mathrm{e}^{-t/5} = 4/5$: $v = 80 - 50 = 30$

b. **$t = 5\ln 2$ s**

- $v = 0$ when $\mathrm{e}^{-t/5} = 1/2$
- $t = 5\ln 2$

c. **$250 - 250\ln 2$ m**

- $\int_0^{5\ln 2} (100\mathrm{e}^{-t/5} - 50)\,\mathrm{d}t = [-500\mathrm{e}^{-t/5} - 50t]$
- $= (-250 - 250\ln 2) - (-500) = 250 - 250\ln 2$

Task B: Catherine wheel

a. **3 m/s**

- $\mathbf{v} = -3\sin t\,\mathbf{i} + 3\cos t\,\mathbf{j}$, with $|\mathbf{v}| = 3$

b. **3 m/s²**

- $\mathbf{a} = -3\cos t\,\mathbf{i} - 3\sin t\,\mathbf{j}$, with $|\mathbf{a}| = 3$

c. **(−3, 3)**

- At $t = \pi/2$ it is at (0, 3) with velocity (−3, 0).
- One second later it is at (0 − 3, 3) = (−3, 3)

Extension

1. **440**
   - Inductive step: $k(k + 1)(k + 2)/3 + (k + 1)(k + 2) = (k + 1)(k + 2)(k + 3)/3$.
   - $n = 10$: $10 \times 11 \times 12 \div 3 = 440$

Every question is original and every answer was checked before publishing.