Differentiation questions and answers

11 questions · Extended · 10 non-calculator · worked answers

Name _____ Date _____ Score _____

- Original questions in the style of the Cambridge 0580 papers, each labelled Core or Extended and calculator or non-calculator. Try every question before you look at the answers.
- Worked answers start after the questions. Questions with marks come from our 0580 practice bank (M method, A accuracy, B independent marks).
- Online, with more practice: igcsemathrevision.com/igcse-frontend/0580/differentiation-questions

Extended: for Papers 2 and 4

1. **Extended · non-calculator · easy**

$y = 5x^3$. Find $\frac{dy}{dx}$.

2. **Extended · non-calculator · easy**

$y = 4x^2 - 3x + 7$. Find $\frac{dy}{dx}$.

3. **Extended · non-calculator · easy · 2 marks**

Find $\dfrac{dy}{dx}$ when $y = 2x^5 - 3x^2 + 8$.

4. **Extended · non-calculator · medium**

Find the gradient of the curve $y = x^3 - 2x$ at the point where $x = 2$.

5. **Extended · non-calculator · medium**

The gradient of the curve $y = x^2 - 6x + 1$ is 4 at the point P.
Find the x-coordinate of P.

6. **Extended · non-calculator · medium**

Find the coordinates of the turning point of $y = x^2 - 8x + 3$, and say whether it is a maximum or a minimum.

7. **Extended · non-calculator · medium · 3 marks**

Find the coordinates of the point on the curve $y = x^2 + 4x - 3$ where the gradient is -2.

8. **Extended · non-calculator · hard**

Find the coordinates of the two turning points of $y = x^3 - 3x^2 - 9x + 2$ and determine which is a maximum.

9. **Extended · non-calculator · hard**

Find the equation of the tangent to $y = x^2 + 3x$ at the point where $x = 1$.

10. **Extended · calculator · hard**

A rectangle has a perimeter of 40 cm and a width of x cm, so its area is $A = x(20 - x)$ cm².
Use differentiation to find the greatest possible area.

11. **Extended · non-calculator · hard · 5 marks**

The curve $y = 2x^3 - 9x^2 + 12x$ has two turning points.

Find their coordinates and decide whether each one is a maximum or a minimum.

Answers and worked solutions

Question 1

Answer: $\frac{dy}{dx} = 15x^2$

1. Multiply by the power and reduce the power by 1: $5 \times 3x^2$.

Question 2

Answer: $\frac{dy}{dx} = 8x - 3$

1. $4x^2 \to 8x$, $-3x \to -3$, and the constant $7 \to 0$.

Question 3 — 2 marks

Answer: $10x^4 - 6x$

B2 $10x^4 - 6x$ (B1 for one correct non-zero term)

Question 4

Answer: 10

1. $\frac{dy}{dx} = 3x^2 - 2$
2. At $x = 2$: $12 - 2 = 10$.

Question 5

Answer: $x = 5$

1. $\frac{dy}{dx} = 2x - 6$
2. $2x - 6 = 4$, so $x = 5$.

Question 6

Answer: $(4, -13)$, minimum

1. $\frac{dy}{dx} = 2x - 8 = 0$, so $x = 4$.
2. $y = 16 - 32 + 3 = -13$.
3. The x^2 term is positive, so the curve is U-shaped: a minimum.

Question 7 — 3 marks

Answer: $(-3, -6)$

M1 $\frac{dy}{dx} = 2x + 4$

M1 $2x + 4 = -2$, $x = -3$

A1 $(-3, -6)$

Question 8

Answer: $(-1, 7)$ maximum, $(3, -25)$ minimum

1. $\frac{dy}{dx} = 3x^2 - 6x - 9 = 3(x - 3)(x + 1)$, so $x = 3$ or $x = -1$.
2. $x = -1$: $y = -1 - 3 + 9 + 2 = 7$. $x = 3$: $y = 27 - 27 - 27 + 2 = -25$.
3. A positive cubic rises, falls, then rises, so the first turning point is the maximum.

Question 9

Answer: $y = 5x - 1$

1. At $x = 1$, $y = 4$. $\frac{dy}{dx} = 2x + 3 = 5$.
2. $y - 4 = 5(x - 1)$, so $y = 5x - 1$.

Question 10

Answer: $100\,\text{cm}^2$

1. $A = 20x - x^2$, so $\frac{dA}{dx} = 20 - 2x$.
2. $20 - 2x = 0$ gives $x = 10$.
   $A = 10 \times 10 = 100$.

Question 11 — 5 marks

Answer: $(1, 5)$ maximum, $(2, 4)$ minimum

M1 $\frac{dy}{dx} = 6x^2 - 18x + 12$

M1 $6(x - 1)(x - 2) = 0$, $x = 1$ or $x = 2$

A1 (1, 5) and (2, 4)

M1 a correct method, e.g. $\frac{d^2y}{dx^2} = 12x - 18$, or the gradient either side

A1 (1, 5) maximum, (2, 4) minimum

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