Proof: exam questions

10 questions · 27 marks · mark schemes at the end

Name _____ Date _____ Score / 27

- Calculators are allowed on both 4MA1 Higher papers (1H and 2H). Show your working: most marks are for method.
- Mark schemes start after the questions. M marks are for a correct method, A marks for an accurate answer that follows a correct method, and B marks for a correct result on its own.
- Online, with more practice: igcsemathrevision.com/igcse-frontend/4ma1/exam-questions/proof

1. **2 marks**

Prove that the sum of any three consecutive integers is a multiple of 3.

2. **3 marks**

Prove that $(2n+1)^2 - (2n-1)^2$ is a multiple of 8 for all integers n.

3. **3 marks**

Prove that the product of any two odd numbers is odd.

4. **2 marks**

Show that $(n+5)^2 - (n+3)^2 = 4(n+4)$.

5. **3 marks**

Prove that the difference between the squares of two consecutive even numbers is a multiple of 4.

6. **4 marks**

Prove that the square of any odd number is one more than a multiple of 8.

7. **2 marks**

Ali says, "$n^2 + n + 11$ is a prime number for every positive integer n."

Show that Ali is wrong.

8. **2 marks**

Show that $3(x+4)-2(x-1)=x+14$ for all values of x.

9. **3 marks**

Prove that the sum of the squares of any two consecutive integers is odd.

10. **3 marks**

Prove that $(n+1)^3-n^3$ is odd for all positive integers n.

Answers and worked solutions

Question 1 — 2 marks

M1 $n + (n+1) + (n+2) = 3n + 3$

A1 $= 3(n+1)$, which is a multiple of 3

Answer: proof

Question 2 — 3 marks

M1 $4n^2 + 4n + 1$ or $4n^2 - 4n + 1$

M1 $4n^2 + 4n + 1 - (4n^2 - 4n + 1)$

A1 $8n$, which is a multiple of 8

Answer: proof

Question 3 — 3 marks

M1 $(2m+1)(2n+1)$

M1 $4mn + 2m + 2n + 1$

A1 $2(2mn + m + n) + 1$, which is one more than an even number, so odd

Answer: proof

Question 4 — 2 marks

M1 $n^2 + 10n + 25 - (n^2 + 6n + 9)$

A1 $4n + 16 = 4(n+4)$

Answer: shown

Question 5 — 3 marks

M1 $(2n+2)^2 - (2n)^2$

M1 $4n^2 + 8n + 4 - 4n^2 = 8n + 4$

A1 $4(2n+1)$, a multiple of 4

Answer: proof

Question 6 — 4 marks

M1 $(2n+1)^2 = 4n^2 + 4n + 1$

M1 $4n(n+1) + 1$

M1 one of n and $n+1$ is even, so $n(n+1)$ is even

A1 $4n(n+1)$ is a multiple of 8, so the square is one more than a multiple of 8

Answer: proof

Question 7 — 2 marks

M1 a value of n tried that gives a non-prime, e.g. $n = 10$

A1 $10^2 + 10 + 11 = 121 = 11 \times 11$, which is not prime

Answer: $n = 10$ gives 121, not prime

Question 8 — 2 marks

M1 $3x + 12 - 2x + 2$

A1 $x + 14$ shown

Answer: shown

Question 9 — 3 marks

M1 $n^2 + (n+1)^2$

M1 $2n^2 + 2n + 1$

A1 $2(n^2 + n) + 1$, which is odd

Answer: proof

Question 10 — 3 marks

M1 $n^3 + 3n^2 + 3n + 1 - n^3 = 3n^2 + 3n + 1$

M1 $3n(n+1) + 1$, and $n(n+1)$ is even

A1 $3n(n+1)$ is even, so $3n(n+1) + 1$ is odd

Answer: proof

Online, with every solution: igcsemathrevision.com/igcse-frontend/4ma1/exam-questions/proof
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