Differentiation: exam questions

10 questions · 37 marks · mark schemes at the end

Name _____ Date _____ Score / 37

- Calculators are allowed on both 4MA1 Higher papers (1H and 2H). Show your working: most marks are for method.
- Mark schemes start after the questions. M marks are for a correct method, A marks for an accurate answer that follows a correct method, and B marks for a correct result on its own.
- Online, with more practice: igcsemathrevision.com/igcse-frontend/4ma1/exam-questions/differentiation

1. **2 marks**

$y = 4x^3 - 5x^2 + 7x - 2$

Find $\frac{dy}{dx}$.

2. **2 marks**

$y = 3x^2 + \frac{8}{x}$

Find $\frac{dy}{dx}$.

3. **2 marks**

Work out the gradient of the curve $y = x^3 - 4x$ at the point where $x = 2$.

4. **6 marks**

The curve C has equation $y = x^3 - 3x^2 - 9x + 5$.

(a) Find the coordinates of the turning points of C. **(4)**

(b) Determine which turning point is a maximum. Give a reason. **(2)**

5. **4 marks**

Find an equation of the tangent to the curve $y = x^2 - 3x + 4$ at the point where $x = 3$.

6. **4 marks**

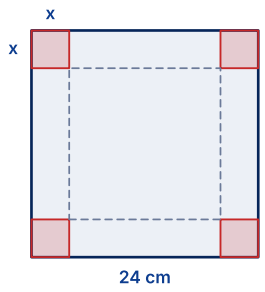
Find an equation of the normal to the curve $y = 2x^2 - 5$ at the point where $x = 1$.

7. **3 marks**

Find the coordinates of the point on the curve $y = x^2 + 5x$ where the gradient is 9.

8. **6 marks**

A square sheet of card has sides of 24 cm. A square of side x cm is cut from each corner and the sides are folded up to make an open box.



(a) Show that the volume of the box is $V = 4x^3 - 96x^2 + 576x$. **(2)**

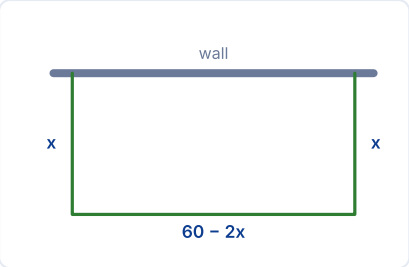
(b) Find the value of x that gives the greatest volume, and work out this volume. **(4)**

9. **4 marks**

Find the coordinates of the points on the curve $y = x^3 - 12x + 1$ where the gradient is zero.

10. **4 marks**

A farmer has 60 m of fencing. He uses it for three sides of a rectangular pen; a wall forms the fourth side. The two sides at right angles to the wall are each x m long.



(a) Show that the area of the pen is $A = 60x - 2x^2$. **(1)**

(b) Find the greatest possible area of the pen. **(3)**

Answers and worked solutions

Question 1 — 2 marks

B2 $12x^2 - 10x + 7$

B1 for two of the three terms correct.

Answer: $12x^2 - 10x + 7$

Question 2 — 2 marks

M1 $\frac{8}{x} = 8x^{-1}$

A1 $6x - \frac{8}{x^2}$

Answer: $6x - \frac{8}{x^2}$

Question 3 — 2 marks

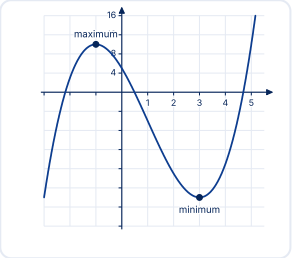
M1 $\frac{dy}{dx} = 3x^2 - 4$

A1 8

Answer: 8

Question 4 — 6 marks

(a)



M1 $\frac{dy}{dx} = 3x^2 - 6x - 9$

M1 $3x^2 - 6x - 9 = 0$, $(x - 3)(x + 1) = 0$

A1 $x = 3$ and $x = -1$

A1 $(3, -22)$ and $(-1, 10)$

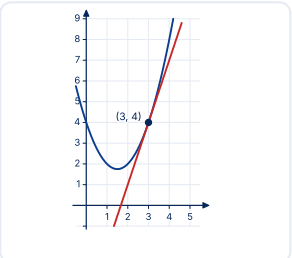
(b)

M1 $\frac{d^2y}{dx^2} = 6x - 6$, or gradients either side

A1 $(-1, 10)$ is a maximum, because $\frac{d^2y}{dx^2} = -12 < 0$ there

Answer: (a) $(3, -22)$ and $(-1, 10)$; (b) $(-1, 10)$

Question 5 — 4 marks



M1 $\frac{dy}{dx} = 2x - 3$

M1 gradient 3 at $x = 3$

M1 point $(3, 4)$, $4 = 3 \times 3 + c$

A1 $y = 3x - 5$

Answer: $y = 3x - 5$

Question 6 — 4 marks

M1 $\frac{dy}{dx} = 4x$, so the gradient of the tangent is 4

M1 gradient of the normal $-\frac{1}{4}$

M1 point $(1, -3)$

A1 $y = -\frac{1}{4}x - \frac{11}{4}$

Accept $x + 4y + 11 = 0$.

Answer: $y = -\frac{1}{4}x - \frac{11}{4}$

Question 7 — 3 marks

M1 $\frac{dy}{dx} = 2x + 5$

M1 $2x + 5 = 9$

A1 $(2, 14)$

Answer: $(2, 14)$

Question 8 — 6 marks

(a)

M1 $V = x(24 - 2x)^2$

A1 $x(576 - 96x + 4x^2) = 4x^3 - 96x^2 + 576x$ shown

(b)

M1 $\frac{dV}{dx} = 12x^2 - 192x + 576$

M1 $12(x - 4)(x - 12) = 0$

A1 $x = 4$ ($x = 12$ gives no box)

A1 $V = 1024\ \text{cm}^3$

Answer: (a) shown; (b) $x = 4$, $1024\ \text{cm}^3$

Question 9 — 4 marks

M1 $\frac{dy}{dx} = 3x^2 - 12$

M1 $3x^2 - 12 = 0$

A1 $x = 2$ or $x = -2$

A1 $(2, -15)$ and $(-2, 17)$

Answer: $(2, -15)$ and $(-2, 17)$

Question 10 — 4 marks

(a)

B1 the side parallel to the wall is $60 - 2x$, so $A = x(60 - 2x) = 60x - 2x^2$

(b)

M1 $\dfrac{dA}{dx} = 60 - 4x$

M1 $60 - 4x = 0$, $x = 15$

A1 $450\,\text{m}^2$

Answer: (a) shown; (b) $450\,\text{m}^2$

Online, with every solution: igcsemathrevision.com/igcse-frontend/4ma1/exam-questions/differentiation
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